Are We Normal?

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

The students will devise two quantitative questions to present to their classmates in a survey. The questions will request information from their peers: one question should require a measurement and the other should necessitate a whole number as a response. The students will write thoughtful analysis by making inferences and investigating the distributions of the data sets. The project will culminate in a presentation of two posters, one for each question, to share the findings. Students will create a histogram and a box-and-whisker plot to use as visual aids.

II. UNIT AUTHOR:

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III. COURSE:

Algebra I

IV. CONTENT STRAND:

Data Analysis

V. OBJECTIVES:

The learner will be able to…

- Collect and organize data
- Construct a box-and-whisker plot
- Construct a histogram
- Calculate and interpret mean absolute deviation and standard deviation

VI. REFERENCE/RESOURCE MATERIALS:

- Graphing calculator
- Student survey sheets (see attached)
- Computer access (optional)

VII. PRIMARY ASSESSMENT STRATEGIES:

Students will be assessed on how accurately their calculations and graphs are done. They will also be evaluated on the thoroughness of their analysis as given in their presentations. Therefore, students need to go beyond reading out the calculations; they need to make inferences from the data, explain the distributions of the graphs, and describe the meaning of the values of the standard deviation or mean absolute deviation. They should be able to defend why they chose to find certain calculations and not others. Of course, the answer to the question, "Are we normal?" should also be clear and supported with relevant calculations.

VIII. EVALUATION CRITERIA:

Students will complete self-evaluations at the conclusion of the task. The teacher will also complete an evaluation using a rubric.

IX. INSTRUCTIONAL TIME:

This task will take about six 50-minute periods to complete. A suggested time line:

- 1 period to discuss project outline and to decide on questions
- 1 period to collect data

- 1 period to organize the data and make initial calculations
- 1 period to draw rough drafts of the graphs and write rough drafts of the analysis
- 1 period to make final copies of the graphs and analysis and prepare for presentations
- 1 period for presentations

Data Collection & Analysis

Strand
Data Analysis

Mathematical Objective(s)
The goal for this activity is to ensure that students can provide insightful and accurate analysis on a data set. The students must also be able to construct corresponding graphs.

Related SOL
- A.9 - The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores
- A.10 - The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

NCTM Standards

- Draw reasonable conclusions about a situation being modeled.
- Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each;
- Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments;
- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable;
- Understand histograms, parallel box plots, and scatterplots and use them to display data;
- Compute basic statistics and understand the distinction between a statistic and a parameter.
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics;

Materials/Resources
- Graphing calculator
- Meter sticks or rulers, as needed
- String, as needed
- Surveys generated by students
- Computer access (optional)

Assumption of Prior Knowledge
- Students should be able to operate the graphing calculator
- Students should be able to calculate basic measures of center and variation such as mean, median, mode, range, quartiles, and extremes
- Students should be able to recognize outliers in a data set
- Students may have difficulty setting up a histogram that follows the normal distribution
- Students may have difficulty making connections from their calculations and graphs to analyze

Introduction: Setting Up the Mathematical Task

- After learning about the idea of a "normal" distribution in relation to z-scores and standard deviation, students will investigate how normal they and their peers may be.
- The question will be posed: "How normal do you think you are?"
- The teacher may prompt students if they need ideas – what kinds of numerical data may students share? It may relate to a quantity of possessions, financial opinions, various measurements, etc.
- After collecting and organizing the data, students will find various calculations and develop graphs to represent their data. They will analyze their results, which will be shared publicly through a short presentation.

Day 1: Question Design

- Before giving students time to discuss what questions they'd like to ask their peers, the teacher should lead a discussion on how to phrase the questions. The teacher may present survey questions that use biased language and ask students to make critiques. Also, it may be necessary to have a discussion about what sorts of questions are not school appropriate.
- In this activity, students will investigate the distribution of data amongst their classmates. They will devise two questions to present in a survey: one must require a whole number response, such as "How many pets do you have?" and the other must require a measurement, such as "How tall are you in centimeters?"
- Students will be given one class period to come up with the two questions. They must get the questions approved by their teacher, which will prevent students from using the same questions. If a class is large, the teacher may decide to pair students together. The students must submit their questions by the end of class. The questions will be handed in on the half-sheet of paper provided, "Are We Normal? A Statistics Investigation" (see attachment, page 7).
- The teacher will type up all of the questions on one page to hand out as a survey, organized by group (see attachment, page 11).

Day 2: Data Collection

- In this activity, the surveys will be passed out to all students in the class. Students will be asked to honestly answer each question about themselves and to be sure to only write one number down for each answer. They may not use a range, e.g. 3 to 5 sodas per week. If students are not sure about an answer, they will have to make their best estimate.
- Students may need assistance with measuring. They may need rulers to find out certain information. For something like the measurement around a fist, string can be used to wrap around it and then measured.
- Although the answers may be anonymous, the teacher should ensure that students are answering honestly and are not making guesses, particularly with the measurements. Any odd data collected will throw off the calculations.
- After students have answered all of the survey questions, they will need to cut out their responses such that each group's questions are separated. All responses will be sorted into the correct group.

Day 3: Data Organization and Calculations

- Each group will receive their responses from the surveys.
- The first goal for each group should be to organize their data. The teacher may suggest that students order their data from least to greatest, but students may decide on other ways.
- With the end goal of making a box-and-whisker plot for question #1 and a histogram, using a backdrop of a normal distribution curve, for question #2, students will need to decide which calculations will be necessary.

- Students will use the graphing calculators to assist in finding any useful calculations. The teacher may need to remind students that the goal of the project is to decide if their classmates are "normal," so they need to have evidence. The analysis needs to be supported with calculations.
- Students may need assistance using the statistics function of their calculators to find mean, median, mode, the quartiles, mean absolute deviation, variance, and/or standard deviation.

Day 4: Rough Drafts of Graphs and Analysis

- Students will create their rough drafts of each graph, using their calculations from the previous day.
- The 1st question in each group will be visualized as a box-and-whisker plot.
- The 2nd question in each group will be visualized with a histogram against a normal distribution curve. Students will be able to compare their histograms to the bell curve to see how the data compares to "normal." Templates of the normal distribution curve are provided (see attachment, page 9).
- Students will write thoughtful commentary about their data, using the questions provided as a guide (see attachment, page 8).
- Both rough drafts should be reviewed by a peer group or the teacher. Suggestions and corrections should be made before completing the final copy.

Day 5: Final Copies of Graphs and Analysis

- Using feedback from the previous day, students will create their final copies on posters or clean computer paper.
- The graphs should be drawn neatly, using straightedges when necessary, and ensuring that all proper components are labeled (title, axes).
- The graph for the 2nd question in each group will be easier to make on a template (see attachment, page 9).
- The analysis should offer information about the calculations and what they suggest about the class as a whole.
- Students, if in groups, will discuss who will be responsible for which sections of the presentation.
- If available, students may create their posters on the computer, using spreadsheet or presentation software.

Day 6: Presentations

- Each group will present both posters along with the corresponding analysis. Presentations, if done thoroughly, will take between 5 and 10 minutes.
- Students will grade themselves using the rubric (see attachment, page 10).
- The teacher will grade the project using the rubric (see attachment, page 10).

Student Exploration

Individual Work / Small Group Work

- The teacher may decide, based on the class size and/or ability on whether or not to group students for the duration of the project.
- Individuals or groups will work together to write two questions that will be used to collect data on their classmates. The questions must require numerical responses; one must be a measurement.
- The data will be analyzed, using measures learned in Algebra I such as mean absolute deviation, standard deviation, and variance.
- Students will create both a histogram and a box-and-whisker plot in order to represent their data.

Student/Teacher Actions:

- Students will be collecting, organizing, and analyzing data resulting from surveys they have written. They will also be creating graphs and preparing a presentation to discuss results from the questions.

- The teacher will be approving the questions and ensuring that none are repeated. The teacher will also proofread the rough drafts and offer constructive criticism to improve the graphs or analysis. Students may need prompting to make the connections between their calculations and the meaning of them.
- The teacher will grade the project using the rubric.
- If available, students may use spreadsheet software to assist in creating the final copies of their graphs.

Monitoring Student Responses

- Students will communicate their thinking by writing their analysis in complete sentences. They will present their findings in class, using their posters as visual aids.
- Students may work together, with teacher permission, to help each other complete each step of the project.
- Students will use the formulas and strategies used within their statistics unit of Algebra I. They will be able to use the statistics functions of their graphing calculators to engage with relevant real-world data.
- If students have difficulties, the teacher can guide them by asking questions. The teacher may need to refer to notes, a textbook, or a website to help students complete all of the activities.
- To extend the activity, students in different classes may collaborate and make comparisons between their data. Students can create parallel box-and-whisker plots to compare data between different classes or subsections.
- The presentations will serve as the closure of the activity. Students will be sharing their results.
- The posters can be collected to review upon completing the rubric if necessary.

Assessment List and Benchmarks

- Students will be assessed on the quality of their posters and presentations. The posters will be checked for accuracy, neatness, and completeness of both the graphs and the analysis paragraphs. The expectations are detailed on the rubric. Teachers can decide how much weight the project will hold in the students' final grades.

Name(s): _____ Group #_____

Are We Normal?
A Statistics Investigation

Please write an appropriate question to pose to your peers that can be answered with one whole number

_____

Please write an appropriate question to pose to your peers that can be answered with a numerical measurement. Specify the units you are expecting.

_____

Name(s): _____ Group #_____

Are We Normal?
A Statistics Investigation

Please write an appropriate question to pose to your peers that can be answered with one whole number

_____

Please write an appropriate question to pose to your peers that can be answered with a numerical measurement. Specify the units you are expecting.

_____

Name(s): _____ Group #_____

Are We Normal?
Data Organization and Analysis

Question #1

The question posed to peers: _____

Write intelligently, in full sentences, about what the graph says about your population. Consider these questions:

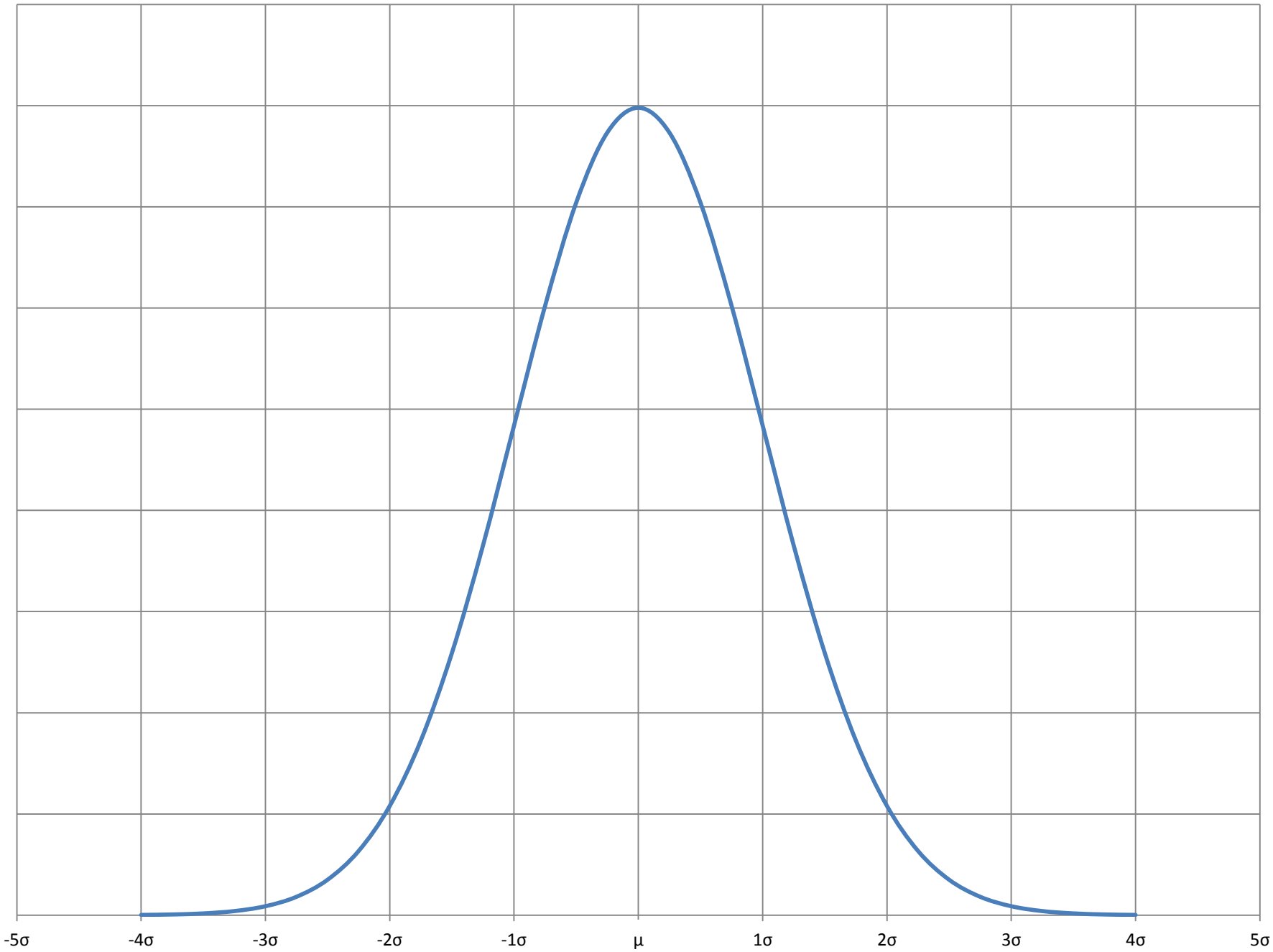
- What does the graph say about your data overall? ("Most people…," "There was / was not a lot of overlap…," etc.)
- Did anything surprise you? ("It was unusual that…")
- Reflecting on the mean absolute deviation, does the data appear to be spread out from the mean or close to it? What does that imply about your classmates? ("Most students are / are not near the mean...")
- How does each of the four sections break down for your graph? Are all four sections roughly the same size or are they very different? What does that say about your data?

Question #2

The question posed to peers: _____

Write intelligently, in full sentences, about what the graph says about your population. Consider these questions:

- What does the graph say about your data overall? ("Most people…," "There was / was not a lot of overlap…," etc.)
- Did anything surprise you? ("It was unusual that…")
- Reflecting on the standard deviation, does the data appear to be spread out from the mean or close to it? What does that imply about your classmates? ("Most students are / are not near the mean...")
- Does your data follow the standard normal distribution curve?
- What percent of your data is within one standard deviation from the mean?
- What percent of your data is within two standard deviations from the mean?
- What percent of your data is within three standard deviations from the mean?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Are We Normal?

Rubric

Category	2	1	0	Self-Rating	Teacher Rating
Presentation	Both graphs and their analyses were shared in an appropriate manner.	Either the graphs or the analyses were incompletely shared.	Neither graphs nor their analyses were shared in an appropriate manner.		
Graph (Question 1)	The graph is accurate, neat, and legible.	The graph is accurate but may be difficult to read at times.	The graph is neither accurate nor neat.		
Analysis (Question 1)	The discussion of how the graph represents the data is comprehensive and accurate.	The discussion of how the graph represents the data is only partially complete.	The discussion of how the graph represents the data not complete.		
Calculations (Question 1)	The calculations shared are accurate and appropriate for the analysis.	The calculations shared are either not appropriate or are not accurate.	No calculations were shared in defense of the analysis.		
Graph (Question 2)	Both graphs and their analyses were shared in an appropriate manner.	Either the graphs or the analyses were incompletely shared.	Neither graphs nor their analyses were shared in an appropriate manner.		
Analysis (Question 2)	The graph is accurate, neat, and legible.	The graph is accurate but may be difficult to read at times.	The graph is neither accurate nor neat.		
Calculations (Question 2)	The discussion of how the graph represents the data is comprehensive and accurate.	The discussion of how the graph represents the data is only partially complete.	The discussion of how the graph represents the data not complete.		

Total: _____ / 14

Comments:

Example of Survey Questions

Are We Normal?

Student Survey

Your answers will remain anonymous. Please answer each question with only one response.

#1	#2
1) How many texts did you send yesterday? 2) How tall are you in centimeters?	1) How many hours do you spend playing or practicing sports in a week? 2) How many centimeters high can you jump?
#3	**#4**
1) How many sodas do you drink in one week? 2) How many centimeters long is your foot?	1) How many letters are in your full legal name? 2) How many beats per minute is your resting heart rate?
#5	**#6**
1) How many pets are in your household? 2) How many centimeters around is your head?	1) How many times per day do you ask to get water or go to the bathroom in order to get out of class? 2) How old are you in days?

Benchmarks

Question 1: How many phones (cellular and landline) are in your household?

Mean (μ): 4.9

Median: 4.5

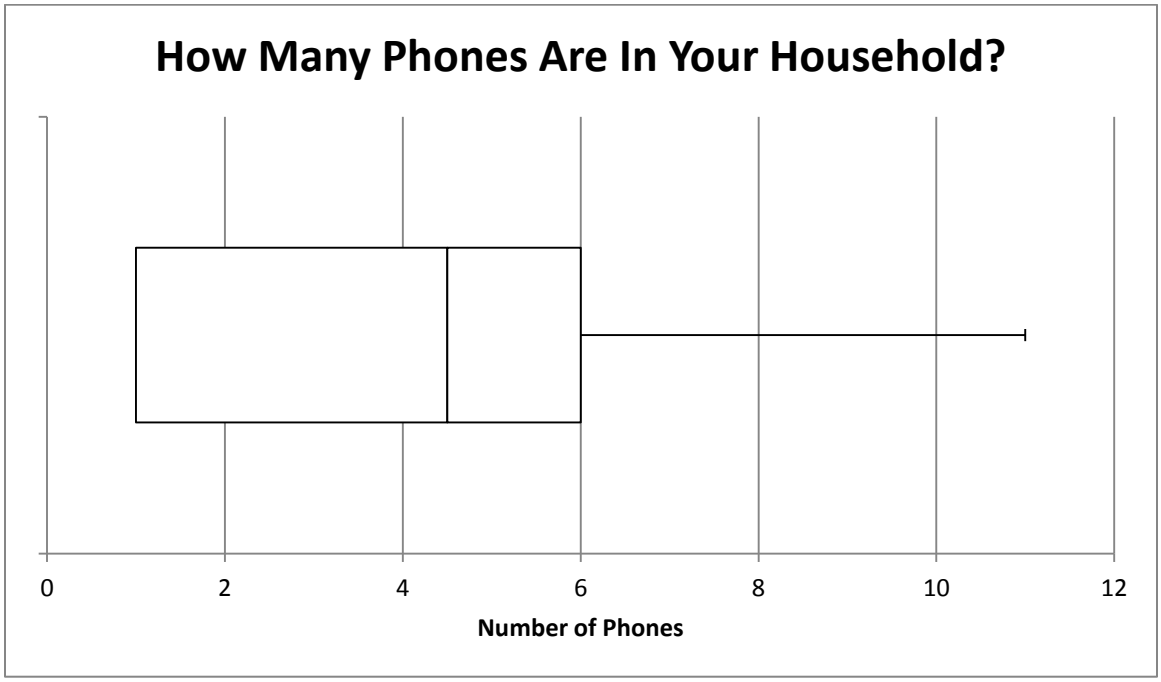
Mode: 1

Range: 11 – 1 = 10

Lower Quartile: 1

Upper Quartile: 6

Interquartile Range: 6 – 1 = 5

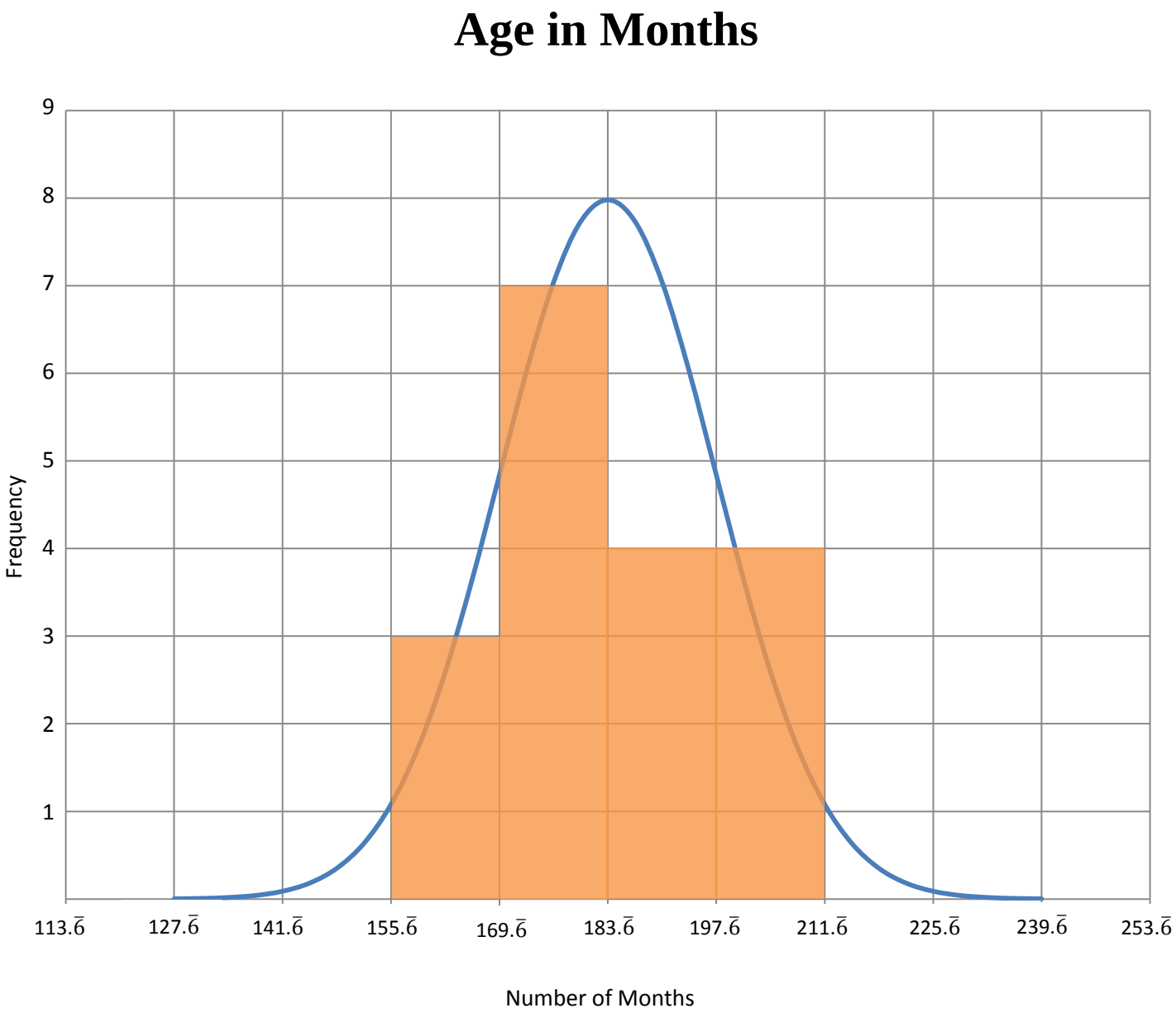


The most common number of phones in a household is one. About 25% of the students have one phone in their household, which is why the lower extreme and the lower quartile are the same. The data is more spread out throughout the rest of the box-and-whisker plot. About 50% of the students have between 1 and 6 phones in their household. The last 25% has a similar range, from 6 to 11 phones. Although it seems like 11 phones is a lot compared to the rest of the data, it still does not constitute itself as an outlier. It is likely that the households with more phones are also the households with more people. Most households have 6 or fewer phones, which makes sense because most households do not have more than 6 people. The mean absolute deviation, 1.7, indicates that there is not a lot of variation with respect to the mean, 4.9.

Question 2: How old are you in months?

Data: 168, 169, 169, 170, 171, 171, 175, 177, 182, 183, 185, 185, 185, 190, 200, 208, 208, 210

Mean: $\frac{551}{3} = 183.\overline{6}$ Standard deviation: $\frac{2\sqrt{437}}{3} \approx 14$



The average age in the class is $183.\overline{6}$ months, or a little over 15 years old. The standard deviation is about 14 months, which is a little more than a year. This makes sense for this class because there are 8^{th}, 9^{th}, and 10^{th} graders enrolled in the class, which means there is more variety in the ages than if students were all in the same grade. There are 10 students who are younger than the average age and 8 students who are older than the average age. This is due to having more 8^{th} and 9^{th} graders in the class than 10^{th} graders. Since the data is not quite symmetric, it does not follow the normal distribution curve. About 61% of the students fall within one standard deviation of the mean and all 100% of the students fall within 2 standard deviations of the mean.